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Branes wrapping black holes as a purely gravitational dielectric effect

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Abstract: In this paper we give a microscopical description of certain configurations of branes wrapping black hole horizons in terms of dielectric gravitational waves. Interestingly, the configurations are stable only due to the gravitational background. Therefore, this constitutes a nice example of purely gravitational dielectric effect.

Keywords: D-branes, Black Holes in String Theory.

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1. Introduction

Recently, an interesting proposal for the AdS_2/CFT_1 correspondence was raised in [1]. From the expression for the $\mathcal{N}=2$ 4-dimensional black hole partition function in [2], it was observed that the most natural way of describing the black hole is in terms of fixed magnetic charges and a certain ensemble of weighted electric charges. This led to the study of a system of N D0 branes (electric charges) in the background attractor geometry produced by magnetic D4 flux ([3]). Putting all the observations together, it was conjectured in [1] that the superconformal quantum mechanics of D0 branes is the dual of string theory in the attractor geometry, which is $AdS_2 \times S^2 \times \mathcal{M}$, being the horizon of the black hole the S^2 and $\mathcal{M}$ a certain compact manifold (which must be Calabi-Yau if we want $\mathcal{N}=2$ SUSY).

In this background, there are several stable supersymmetric brane configurations which are wrapping non-contractible cycles of $S^2 \times \mathcal{M}$. Among them, it is of particular interest that of a D2 brane wrapping the S^2 with some DBI flux on it, carrying momentum along $\mathcal{M}$. As it was shown in [4], these configurations preserve one half of the supersymmetries, and, since they do not have net D2 charge, they contribute only to D0 charge. Indeed, in view of the Myers effect ([5]), one can understand microscopically these configurations in terms of dielectric D0 branes. Such brane configurations provide a natural understanding of the black hole entropy, since it can be regarded as the degeneracy of the ground state of the configuration, which is a Lowest Landau Level.

In a subsequent paper ([6]) it was realized that similar configurations of branes wrapping horizons with similar dispersion relations occur in many other black hole backgrounds. In this reference, the dynamics of branes wrapping black hole horizons in geometries of the form $AdS_m \times S^n \times \mathcal{M}$ was studied from the point of view of the $DBI+CS$ effective action. The branes are assumed to wrap the sphere and, since in general they carry momentum in $\mathcal{M}$, they couple to the background potential. As in the $AdS_2 \times S^2$ case, these p-branes do not contribute to net p-brane charge, but to D0 (or momentum) charge.

In this note we will concentrate on a class of these configurations,[1] namely the ones static in $\mathcal{M}$ (as we will see, due to this ansatz, the considered branes do not couple to the background potential). This ansatz implies that our branes lie in the lowest energy level, which is a Lowest Landau Level. In particular, we will work with the 11 dimensional $AdS_3 \times S^2 \times \mathcal{M}$ background, which can be seen as the near horizon geometry of the 11d black string, which is carrying momentum along a certain direction y. This background leads, upon reduction along y, to a type-IIA geometry whose near horizon is precisely $AdS_2 \times S^2 \times \mathcal{M}$. We will also study the type-IIB $AdS_3 \times S^3 \times \mathcal{M}$ background.

In both the M-theory and type-IIB setups we will have stable expanded p-branes which do not carry p-brane charge but momentum charge along a direction cointained in the AdS. In a sense, this is very reminiscent of the giant gravitons ([7]). Giant gravitons are expanded stable branes wrapping a contractible cycle and orbiting in the background. Since the coupling to the background flux compensates exactly the tension of the brane, they behave as a massless particle. Given that these branes carry momentum charge dissolved in their worldvolume, in the view of the Myers effect ([5]), it is natural to conjecture that there should exist a microscopical description of giant gravitons in terms of pointlike gravitons (gravitational waves) expanded due to dielectric effect. In very much of the same spirit of this, in this note we will see that we can give a microscopical description of these configurations using the action for coincident gravitons given in [8].[2]

As we have so far mentioned, due to the fact that the branes we will consider are static in $\mathcal{M}$, in both the 11d and IIB case the configuration is stable with no help of the form potential. In other words, this provides an example of purely gravitational dielectric effect ([10]).

[1] For further details about the properties of these configurations, such as the preserved supersymmetries and other interesting issues, we refer to [6].

[2] A microscopical description for a certain class of giant gravitons has been also discused from a slightly different point of view also in [9].

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Purely gravitational dielectric effect was expected in [10] for time dependent configurations in non-trivial backgrounds.[3] In this reference it is also pointed that this effect could be of importance in the context of black holes. Here we have a nice example of this, since in the type-IIB and M-theory cases the configurations we are dealing with carry momentum in a certain direction (i.e. are not static) and are stable only due to the metric background, with no help of the fluxes.

The type-IIA case appears, at first sight, somehow different, since we will have to consider not gravitons but static (in global coordinates) D0 branes. We will see that this can be understood from its 11d origin. Due to the assumption that the configuration sits in the origin[4] of $\mathcal{M}$, there is no coupling to the RR 3-form potential. In turn, there is only the monopolar coupling of each D0 brane to the RR 1-form potential. We can gain some understanding by working in the full 11d geometry prior to the near horizon limit, where we will see that there are stable configurations of dielectric gravitational waves moving in y expanded only due to gravitational effects. Upon reduction, the GW become dielectric D0 branes, and consequently these configurations will give rise to their IIA counterparts in terms of D0 branes. From this point of view, the particular IIA configurations we will study are due to nothing but the purely gravitational dielectric effect but seen with the Kaluza-Klein glasses.

Although we will consider the most supersymmetric backgrounds in each situation,[5] since for simplicity we will assume the configurations to be in the origin of the extra manifold, the discussion could be lift to the $\mathcal{N} = 2$ case. In addition, the simplification of considering the configurations at the origin of $\mathcal{M}$ makes pretty obvious that, even in the most general case, the purely gravitational dielectric effect is playing a key role.

2. The action for coincident gravitons

In [12] the first step towards a microscopical description of coincident gravitons was made. The final theory for 11 dimensional coincident gravitons with dielectric couplings in their worldvolume was given in [8]. This action has been successfully used, providing a microscopical description of giant gravitons in a series of papers ([13 − 15]) with a perfect agreement with their macroscopic counterparts in terms of usual branes. More recently, this action was shown to contain as a limit the BMN matrix model, and used to derive a "strongly coupled" version of the BMN matrix model by means of a 9-11 flip, which allowed to give a microscopical description of the M5 vacuum of the 11d Pp-wave ([16]).

The worldvolume theory associated to N coincident gravitational waves in M-theory is a U(N) gauge theory, in which the vector field is associated to M2-branes (wrapped on the direction of propagation of the waves) ending on them [8]. This vector field gives

[3] For some examples of purely gravitatorial dielectric effect see [11].

[4] Strictly speaking, we only need the branes to be static in $\mathcal{M}$. In any case, we can always choose this point to be at the origin.

[5] In general we could consider M-theory in a certain manifold $\mathcal{M}$, whose near horizon is $AdS_3 \times S^2 \times \mathcal{M}$. Upon reduction we would have a geometry whose near horizon would be $AdS_2 \times S^2 \times \mathcal{M}$. If $\mathcal{M}$ were a CY_3 we would have a $\mathcal{N} = 2$ background, while if $\mathcal{M} = T^6$ we have the maximal supersymmetry.

the BI field living in a set of coincident D0-branes upon reduction along the direction of propagation of the waves.

In this paper we will use a truncated version of the action in [8] in which the vector field is set to zero, given that it will not play any role in the backgrounds that we will be discussing. This action is given by $S = S^{BI} + S^{CS}$, with

$$S^{BI} = -T_0 \int dt \ \mathrm{STr}\left\{ k^{-1}\sqrt{-P[E_{00} + E_{0i}(Q^{-1}-\delta)^i_k E^{kj} E_{j0}]\det \mathrm{Q}} \right\} \ , \tag{2.1}$$

where

$$\begin{aligned} &E_{\mu\nu} = \mathcal{G}_{\mu\nu} + k^{-1}(i_k C^{(3)})_{\mu\nu} \ , \qquad \mathcal{G}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \ , \\ &Q^i_j = \delta^i_j + ik[X^i, X^k]E_{kj} \ ; \end{aligned} \tag{2.2}$$

and

$$S^{CS} = T_0 \int dt \, \mathrm{STr}\left\{ -P[k^{-2}k^{(1)}] + iP[(i_X i_X)C^{(3)}] + \frac{1}{2}P[(i_X i_X)^2 i_k C^{(6)}] + \cdots \right\} \ , \tag{2.3}$$

where the dots include couplings to higher order background potentials and products of different background fields contracted with the non-abelian scalars.[6]

In (2.1)+(2.3) k^μ is the abelian Killing vector which, by construction (see [8]), points on the direction of propagation of the waves. This direction is isometric,[7] because the background fields are either contracted with the Killing vector, so that any component along the isometric direction of the contracted field vanishes, or pulled back in the worldvolume with covariant derivatives relative to the isometry (see [8] for their explicit definition).[8] Therefore, the action exhibits a U(1) isometry associated to translations along this direction, which, by construction, is also the direction on which the waves propagate. For more details about the construction of the action, see [12] and [8].

As we have pointed, the action (2.1) + (2.3) has been successfully used in the microscopical study of giant graviton configurations in backgrounds which are not linear perturbations to Minkowski. In all cases perfect agreement with the description of [7] has been found in the limit of large number of gravitons, in which the commutative configurations of [7] become an increasingly better approximation to the non-commutative microscopical configurations, in very much the same spirit as in [5]. Although a number of subtleties can be risen (involving e.g. the symmetrized trace or even the proper form of the action as a low energy effective action), we will take a more pragmatical point of view, and use the action in very much the same spirit as in [13 – 16].

[6]These couplings are not shown explicitly because they will not play a role in the backgrounds under consideration in this paper.

[7]Indeed, the action (2.1)+(2.3) is a gauged σ-model in the spirit of [17 – 20] , which eliminates the dependence in the isometric coordinate.

[8]The reduced metric $\mathcal{G}_{\mu\nu}$ appearing in (2.2) is in fact defined such that its pull-back with ordinary derivatives equals the pull-back of $g_{\mu\nu}$ with these covariant derivatives.

3. The extremal black string in M-theory on T^6

We will concentrate in a certain class of compactifications of M-theory on T^6. We will parametrize the 6-torus with $\{y^i,\ i=1,\dots 6\}$. By placing three stacks of M5 branes with charges $\{p_1,p_2,p_3\}$ carrying momentum q_0 along a direction y and wrapping respectively $\{y,y^3,y^4,y^5,y^6\}$, $\{y,y^1,y^2,y^5,y^6\}$ and $\{y,y^1,y^2,y^3,y^4\}$, we have that the metric is given by

$$ds^2 = h^{-\frac{1}{3}}\left[-dt^2+dy^2+\frac{q_0}{r}(dt-dy)^2\right]+h^{\frac{2}{3}}\left[dr^2+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\right]+h^{-\frac{1}{3}}\Sigma_{i=1,2,3}H_i ds^2_{T_i} \ . \tag{3.1}$$

There is also a 3-form potential, given by

$$C^{(3)} = \sin\theta d\theta\wedge d\phi\wedge\left[p_3\frac{y^5dy^6-y^6dy^5}{2}+p_2\frac{y^3dy^4-y^4dy^3}{2}+p_1\frac{y^1dy^2-y^2dy^1}{2}\right] \ . \tag{3.2}$$

We have defined

$$h=H_1H_2H_3\ ; \qquad H_i = 1+\frac{p_i}{r},\ i=1,2,3\ ; \qquad H_0 = 1+\frac{q_0}{r}\ .$$

The charges p_i of the stacks of branes are related to the number of branes n_i as

$$p_i = \frac{2\pi^2 n_i}{M_{11}^3 vol(T_i)}\ . \tag{3.3}$$

The geometry given by (3.1) and (3.2) is that of a black string, whose horizon is the S^2.

We can take the near horizon limit of this geometry. The way in which this limit is taken is somehow subtle, since it is different depending whether q_0 zero is or not (the details can be seen in, for example, [6]). However, the resulting space is in both cases $AdS_3\times S^2\times T^6$, whose metric in global coordinates is

$$ds^2 = R^2[-\cosh^2\chi d\tau^2+d\chi^2+\sinh^2\chi d\varphi^2]+\tilde{R}^2 d\Omega_2+ds^2_{T^6}(y^i)\ . \tag{3.4}$$

There is also a 3-form potential, given by

$$C^{(3)} = A_i(y^j)d\omega_2\wedge dy^i\ . \tag{3.5}$$

Here $d\omega_2$ in (3.5) stands for the volume form in the 2-sphere.

The parameters R and $\tilde{R}$ are given in terms of $\lambda=(p_1p_2p_3)^{\frac{1}{3}}$ as

$$R=2\lambda\ , \quad \tilde{R}^2=\tfrac{1}{\lambda}\ .$$

3.1 M2 wrapping the horizon

We will consider an M2 brane wrapping the S^2 and moving in the φ direction in the background given by (3.4)+(3.5). We will also take the ansatz that it lives at $y^i=0$. Then, by using the standard $DBI+CS$ effective action for the brane, it is straightforward to see that the action for such a configuration is

$$S=-4\pi T_2\tilde{R}^2R\int d\tau\sqrt{\cosh^2\chi-\sinh^2\chi\dot{\varphi}^2}\ , \tag{3.6}$$

where T_2 stands for the M2 tension.

Since φ does not appear in the action, its canonically conjugated momentum is a conserved quantity. Performing a Legendre transformation, we can switch to the conserved hamiltonian

$$H = \cosh\chi \sqrt{16\pi^2 T_2^2 \tilde{R}^4 R^2 + \frac{P^2}{\sinh^2\chi}} \ . \tag{3.7}$$

Minimizing (3.7) with respect to χ we find a minimum at

$$\sinh^2\chi = \frac{P}{4\pi T_2 \tilde{R}^2 R} \ . \tag{3.8}$$

For this minimum, the energy of the configuration is

$$E = P + 4\pi T_2 \tilde{R}^2 R \ . \tag{3.9}$$

This particular dispersion relation can be related with the particular isometry group of the AdS_3 geometry, as can be seen in [6].

This configuration is a particular case of the ones considered in [6], when we assume the brane to be at the origin of the T^6. Indeed, the results obtained here coincide with the ones in that reference once we assume that there is no velocity in $\mathcal{M}$ (we will go back to this point later). It is important to notice that due to this ansatz there is no contribution from the CS part of the action, since in the ansatz $y^i = 0$ the would-be coupling to the 3-form vanishes. Then, the geometry is stable only due to the metric background. Notice that if the brane had zero momentum in φ, it would be sitting in the center of the AdS, as one can see from (3.8). In a sense, the "centrifugal force" is pulling the configuration out of the center against the AdS throat.

3.2 Microscopical description of the M2 wrapping the horizon

In order to give a microscopical description of the M2 brane wrapping the horizon of the previous subsection, we will use the action for 11 dimensional coincident gravitons.

Since we are taking the configuration to be at the origin of the torus, we see that there is no contribution from the 3-form potential, which vanishes for constant y^i. Therefore the only contribution to the action will be that of the DBI, given by (2.1), in which the coupling to the 3-form potential is zero.

Writing the effective background in suitable cartesian coordinates it reads

$$ds^2 = R^2[-\cosh^2\chi d\tau^2 + d\chi^2 + \sinh^2 d\varphi^2] + \tilde{R}^2(dx^i)^2 \ , \tag{3.10}$$

where $\{x^i \mid \Sigma(x^i)^2 = 1\}$ are the cartesian coordinates of the S^2.

Since our macroscopic M2 was moving in the φ direction, we should take in the microscopic description φ as the direction of propagation of the gravitational waves, i.e. $k^\mu = \delta^\mu_\varphi \to k^2 = R^2 \sinh^2\chi$.

The fuzzy manifold to which the gravitons will expand will be the S^2, for which we will take the non-commutative S^2 ansatz

$$X^i = \frac{1}{\sqrt{C_2}} J^i \ , \tag{3.11}$$

where J^i are the SU(2) generators in an N-dimensional fundamental representation of SU(2) whose quadratic Casimir is C_2. Therefore they satisfy $\Sigma(X^i)^2 = 1$ as a matrix identity.

By substituting this ansatz into (2.1) we have that the Q matrix in (2.1) is

$$Q^i_j = \delta^i_j + i\tilde{R}^2 R \sinh\chi [X^i, X^j] \ . \tag{3.12}$$

It is easy to see that the piece in (2.1) involving $(Q^{-1} - 1)$ does not contribute, and therefore the only purely non-abelian contribution is that of $det(Q)$. Whith the help of the symmetrized trace it can be seen that

$$det(Q) = 1 + \frac{4\tilde{R}^4 R^2 \sinh^2\chi}{C_2} \ . \tag{3.13}$$

Since the configuration is effectively static, we can compute the hamiltonian easily as $H = -L$. In addition, expanding the square root in the Myers limit of [5] $\tilde{R} \ll \sqrt{N}$, we can compute the symmetrized trace and regard the expression as the expansion of

$$H = \cosh\chi \sqrt{\frac{4N^2T_0^2\tilde{R}^4R^2}{C_2} + \frac{(NT_0)^2}{\sinh^2\chi}} \tag{3.14}$$

up to higher order corrections in $\frac{R}{N^2}$.

Taking into account that in our conventions $T_0 = 2\pi T_2$, and that by construction $P = NT_0$, it is clear that in the large-N limit when $\frac{N}{C_2} \to 1$ both the expressions (3.7) and (3.14) coincide.

Thus, we see that there is a microscopical description for the M2 brane wrapping the horizon in terms of dielectric gravitons expanded due to dielectric effect. This expansion is caused not by the flux, but by the proper geometry, and therefore it is a nice example of purely gravitational dielectric effect.

Extremizing (3.14) we find that there is a minimum at

$$\sinh^2\chi = \frac{\sqrt{C_2}}{2\tilde{R}^2 R} \ , \tag{3.15}$$

whose energy is

$$E = NT_0 + 2\tilde{R}^2 R T_0 \frac{N}{\sqrt{C_2}} \ . \tag{3.16}$$

In large-N both the position and the value of the energy at the minimum coincide with their macroscopical counterparts given by (3.8) and (3.16). This is to be expected, since the description in terms of gravitational waves is only valid for coincident GW. By taking N large enough this regimen of validity overlaps with that of the macroscopical description, exactly as it was discussed in [5].

It is worth saying a few words on the stability of our solution. On one hand it is wrapping a non-contractible cycle in the background geometry, and therefore it is topologically stable against contraction to a point. Contrary to the giant graviton case, where the radius of the configuration is a modulus which we should put on-shell, now the radius of the configuration is simply the AdS radius, which is fixed. On the other hand, as we

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will show later, the configuration is in a Lowest Landau Level in the internal torus, which means that it is already in the ground state and therefore cannot decay. The only possible instability would be in the position in the AdS, but simply by plotting the energy we see that there is no other extrema for our system that the one found (which is an absolute minimum). Therefore it is clear that the configuration is stable. As we will see, in the IIA case the position of the configuration in the AdS can be even mapped to the condition for the configuration to be one half BPS, which is again pointing to the stability of the configuration.

4. From black strings in M-theory to black holes in IIA

The geometry given by (3.1)+(3.2) can be reduced to a type IIA black hole by performing a Kaluza-Klein reduction along the y direction. The resulting metric is

$$ds^2 = -(H_0 h)^{-\frac{1}{2}} dt^2 + (H_0 h)^{\frac{1}{2}} \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + (\frac{h}{H_0})^{-\frac{1}{2}} \Sigma_{i=1,2,3} H_i ds^2_{T_i} \ . \quad (4.1)$$

There is also a 3-form RR potential given by

$$C^{(3)} = \sin\theta d\theta \wedge d\phi \wedge \left[p_3 \frac{y^5 dy^6 - y^6 dy^5}{2} + p_2 \frac{y^3 dy^4 - y^4 dy^3}{2} + p_1 \frac{y^1 dy^2 - y^2 dy^1}{2}\right] \ . \quad (4.2)$$

Since in 11 dimensions the 3 stacks of M5 branes carry momentum along a worldvolume direction y (which is the one along we reduce), once we reduce we have 4 intersecting stacks of D4 branes which carry a non-zero D0-brane charge (indeed, one can see that if $q_0 = 0$ there is a null singularity at $r = 0$ in IIA) given by a RR 1-form potential

$$C^{(1)} = \left(1 - \frac{1}{H_0}\right) dt \ . \quad (4.3)$$

There is also a non-zero dilaton, given by

$$e^{\Phi} = \frac{H_0^3}{h} \ . \quad (4.4)$$

Again, there is a horizon in $r = 0$. We can take the near horizon limit of the IIA geometry to have[9]

$$\begin{aligned} ds^2 &= R_{IIA}^2\Big[-\cosh^2\chi d\tau^2 + d\chi^2\Big] + R_{IIA}^2\Big[d\theta^2 + \sin^2\theta d\phi^2\Big] + \\ &\quad + \sqrt{\frac{q_0 p_1}{p_2 p_3}}\left((dy^1)^2 + (dy^2)^2\right) + \sqrt{\frac{q_0 p_2}{p_1 p_3}}\left((dy^3)^2 + (dy^4)^2\right) + \\ &\quad + \sqrt{\frac{q_0 p_3}{p_2 p_1}}\left((dy^5)^2 + (dy^6)^2\right), \quad (4.5) \\ C^{(3)} &= \sin\theta d\theta \wedge d\phi \wedge \left[p_3 \frac{y^5 dy^6 - y^6 dy^5}{2} + p_2 \frac{y^3 dy^4 - y^4 dy^3}{2} + p_1 \frac{y^1 dy^2 - y^2 dy^1}{2}\right] , \quad (4.6) \\ C^{(1)} &= -\frac{R_{IIA}^2}{q_0}(1 - \sinh\chi) d\tau \ , \quad (4.7) \\ e^{\Phi} &= \frac{q_0}{R_{IIA}} \ . \quad (4.8) \end{aligned}$$

[9]We also change to global coordinates. The details can be found in [6].

The quantity R_{IIA} appearing in the IIA near horizon limit is defined as $R_{IIA} = (q_0 p_1 p_2 p_3)^{\frac{1}{4}}$.

The geometry given by (4.5)+(4.7)+(4.6)+(4.8) is that of $AdS_2 \times S^2 \times T^6$ in global coordinates. Note that it cannot be obtained as a reduction of the 11 dimensional near horizon geometry.

4.1 D0 branes in the IIA $AdS_2 \times S^2 \times T^6$ geometry

We have obtained the IIA AdS_2 as a dimensional reduction of the corresponding background of M-theory. As we have so far seen, in 11 dimensions there are stable configurations of gravitons expanded to an S^2 in the near horizon limit which are moving along a cycle contained in the AdS_3. When regarded from the point of view of the full geometry instead of just the near-horizon limit, the configuration of gravitons inherits momentum in the y direction. Since we are reducing precisely in this coordinate, we should expect to have stable static configurations of D0 branes wrapping the S^2 as counterparts of the M-theory construction.[10]

4.1.1 M-theory gravitons in the full geometry

Before analyzing the IIA configurations of coincident D0 branes, we will go back to 11 dimensions and analyze coincident gravitons in the full geometry given by (3.1)+(3.2).

In order to understand the IIA configurations from 11d, assume now the particular configuration in which the gravitational waves carry momentum only in the y direction,[11] i.e. $k^\mu = \delta^\mu_y$. Taking the same ansatz as before, it is straightforward to see that the energy for this type of configuration is

$$E = \frac{NT_0}{1+\frac{q_0}{r}} \Big\{ \sqrt{1-\frac{q_0^2}{r^2}} \sqrt{1+\frac{4r^2 h}{C_2}(1+\frac{q_0}{r})} + \frac{q_0}{r} \Big\} \,. \tag{4.9}$$

The last term in (4.9) comes from the monopole term in (2.3). The monopole coupling in the action for gravitational waves represents the fact that gravitons are fundamentally charged with respect to a momentum operator. Since this momentum operator involves a covariant derivative[12] in the gauged σ-model which is the action for gravitational waves, it only contributes when $k_0 + k_i \partial X^i$ (being i different from the direction of propagation) is non-zero. In our particular set-up, $\partial X^i = 0$, but $k_0 = g_{0y} \neq 0$, and therefore there is a non-zero contribution from the monopole term. Notice that once we reduce along y, this monopole coupling to a momentum operator in 11 dimensions gets mapped to a monopole coupling to $C^{(1)}$ in IIA, as expected, since when reducing from M-theory to IIA momentum in the 11th direction becomes D0-brane charge. From this discussion we see that there should exist similar configurations of expanded D0 branes in IIA, which should be now static.

[10] Remember we are assuming static configurations in $\mathcal{M}$ through all the paper. Therefore there will be no coupling to the background potential.

[11] In general there could be momentum in other coordinates, but since we will reduce along y, it will suffice for our purposes with considering momentum in y.

[12] Remember that it reads $k^{-2} k_\mu \mathcal{D} X^\mu$.

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The expression (4.9) is rather complicated as a function of r. In any case, since the energy must be real, we have that $r \geq q_0$. In general, for generic values of $p_1,\ p_2,\ p_3,\ q_0$, we will have stable expanded branes away from the origin.

Interestingly, also in the full geometry we have a configuration of expanded gravitational waves which is stable only due to momentum, with no need of the form potentials. This means that also in the full geometry we have a purely gravitational dielectric effect. Since in the full geometry the waves are moving in the y direction, which is precisely the one along which we are reducing, we should expect analogue configurations in type-IIA, this time in terms of D0 branes. These IIA configurations should be stable not due to 3-form RR potential, but due to 1-form potential, i.e. the mechanism which renders the IIA configuration stable should be the gravitational dielectric effect but seen with KK glasses.

4.1.2 D0 branes in type-IIA

Working in the near-horizon limit of the full type-IIA geometry, we will consider N coincident D0 branes living at the origin of the 6-torus. Therefore we will take $y^i = 0$. Then, there is no coupling of the 3-form RR potential. The effective background for the D0 branes in suitable cartesian coordinates for the S^2 is

$$ds^2 = R_{IIA}^2\Big(- \cosh^2\chi d\tau^2 + d\chi^2\Big) + R_{IIA}^2\Big(dx^2 + dy^2 + dz^2\Big) \ , \tag{4.10}$$

and

$$C_\tau^{(1)} = -\frac{R_{IIA}^2}{q_0}(1 - \sinh\chi) \ . \tag{4.11}$$

Since $\Sigma(x^i)^2 = 1$, we will take the same non-commutative ansatz for the $x^i \to X^i$ as (3.11). Then, upon particularizing the action for coincident D0 branes given in [5], and taking the suitable limit as we did in the M-theory case, we have that the action for the system has a DBI and a CS part which read

$$S_{BDI} = -\frac{NT_0R_{IIA}^2}{q_0}\int d\tau \cosh\chi\sqrt{1 + \frac{4R_{IIA}^4}{C_2}} \ , \tag{4.12}$$

$$S_{CS} = -\frac{NT_0R_{IIA}^2}{q_0}\int d\tau(1 - \sinh\chi) \ . \tag{4.13}$$

Notice that the last term in (4.9) agrees exactly with S_{CS} once we take the near horizon limit. This reflects the anticipated fact that the IIA construction is the KK version of the gravitational dielectric effect.

Since the configuration is static, $H = -L = -L_{DBI} - L_{CS}$:

$$E = \frac{NT_0R_{IIA}^2}{q_0}\left\{\cosh\chi\sqrt{1 + \frac{4R_{IIA}^4}{C_2}} - \sinh\chi + 1\right\} \ . \tag{4.14}$$

Minimizing (4.14) with respect to χ we have a minimum at

$$\tanh\chi = \frac{1}{\sqrt{1 + \frac{4R_{IIA}^4}{C_2}}} \ , \tag{4.15}$$

whose energy is

$$E = \frac{NT_0 R_{IIA}^2}{q_0} \left\{ \frac{2R_{IIA}^2}{\sqrt{C_2}} + 1 \right\} = \frac{NT_0 R_{IIA}^2}{q_0} + \frac{2T_0 R_{IIA}^4}{q_0} \frac{N}{\sqrt{C_2}} \ . \tag{4.16}$$

4.1.3 D2 brane as effective description

We can have an alternative description of the configuration in the previous section in terms of a D2 brane wrapping the horizon. This has been already presented in [6]. We will take static gauge, and assume the brane to be at the origin of the torus. Since the D2 is spherical, it carries no net D2-brane charge. In order to give the suitable D0 brane charge we have to turn on a non-zero DBI vector field, whose field strength is

$$F_{\theta\phi} = \frac{N}{2} \sin\theta \ , \tag{4.17}$$

where N is the total D0 brane charge which the D2 carries.

By using the standard DBI+CS action for a D2 brane in the background we have, it is straightforward to see that the energy for the system (which is $-L$ since the configuration is static) is:

$$E = \frac{2\pi T_2 N R_{IIA}^2}{q_0} \left\{ \cosh\chi \sqrt{1 + \frac{4R_{IIA}^4}{N^2}} - \sinh\chi + 1 \right\} \ . \tag{4.18}$$

Minimizing with respect to χ we have a minimum located at

$$\tanh\chi = \frac{1}{\sqrt{1 + \frac{4R_{IIA}^4}{N^2}}} \ . \tag{4.19}$$

Indeed this condition is precisely the one found in [4] for a supersymmetric D2 brane with DBI flux on it. This means that our microscopical construction is one half BPS at least in the large-N limit, when it overlaps with its macroscopical counterpart. In some sense this implyies the stability of the construction, since on one hand it is clear that it its topologically stable (it wraps a non-contractible cycle in the space), and on the other hand, as we will show later, it is in its ground state in the internal torus (it is a Lowest Landau Level). Therefore it cannot decay. The only possible instability would be that the location in the AdS was not stable, but SUSY ensures now the stability. In any case it is easy to see that there is no other minimum in χ to which the brane could jump (for example simply by plotting the energy one can see immediately that the only extremum is the absolute minimum we found). Therefore we conclude that the configuration is stable.

The energy for this minimum is:

$$E = \frac{2\pi T_2 N R_{IIA}^2}{q_0} + \frac{4\pi T_2 R_{IIA}^4}{q_0} \ . \tag{4.20}$$

Once we take into account that $2\pi T_2 = T_0$ it is straightforward to see that (4.18), (4.19) and (4.20) coincide in the large-N limit (when in the microscopical computation $\frac{N}{\sqrt{C_2}} = 1$) with their microscopical counterparts given by (4.14), (4.15) and (4.16). This coincidence is to be expected precisely only in the large-N limit, since it is only there when we have that both descriptions are valid at the same time, exactly as we have pointed out when discussing the M-theory configuration.

5. IIB black holes

So far we have seen that there is a stable M2 brane wrapping the horizon of the 11 dimensional black string, which can be described microscopically in terms of gravitational waves expanded due to dielectric effect. This dielectric effect is due to purely gravitational effects, since there is no contribution from the 3-form potential. Upon reduction along a direction of propagation, which is related to the one in which the M-theoretic configuration is moving, this gives rise to a type-IIA configuration of expanded D0 branes in which there is no dielectric coupling to a 3-form potential. Its stability is only due to gravitational effects plus the monopolar coupling of each D0 brane to the background 1-form potential. This is a consequence of the fact that the original configuration in 11 dimensions is static only due to gravitational effects.

In this section we will see another example of configuration of brane wrapping the horizon (which has also been studied from the macroscopical point of view in [6]), this time in type-IIB, whose dispersion relation is of the form of the ones we have so far worked out, and which admits a microscopical description in terms of gravitational waves expanded again due to purely gravitational dielectric effect.

We will focus on the type-IIB $AdS_3 \times S^3 \times T^4$ background obtained by taking type-IIB string theory in $T^4 \times S^1$ and wrapping D1 strings on the S^1 and $D5$ branes on the $T^4 \times S^1$. Once we take the near horizon limit we end up with the following $AdS_3 \times S^3 \times T^4$ background (indeed, we will work with a 2 times T-dualized along the torus coordinates y^1, y^2 version of the background, which is then the near horizon geometry of two sets of D3 branes. For further details see [6]):

$$ds^2 = L^2 \left(-\cosh^2\chi d\tau^2 + d\chi^2 + \sinh^2\chi d\varphi^2\right) + L^2 d\Omega_3^2 + ds^2_{T^4} \ . \tag{5.1}$$

In this background there is no dilaton. In addition, the radius of the AdS, L, can be written in terms of the charges of the background. Namely $L^2 = \sqrt{Q_1 Q_2}$.

There is also a 4-form potential which has entries on the T^4, which schematically reads

$$C^{(4)} = \{F_1 d\tau \wedge d\varphi + F_2 d\psi \wedge d\phi + F_3 d\tau \wedge d\psi + F_4 d\varphi \wedge d\phi\} \wedge dy^1 \wedge dy^2 \ . \tag{5.2}$$

Since we will be again taking the ansatz that the configuration is static in T^4, we see that there will be no coupling to the 4-form RR potential.[13] We are thus left only with the metric background given by (5.1).

5.1 The action for IIB gravitational waves

In order to have an action for IIB coincident waves we have to perform a dimensional reduction plus a T-duality from the M-theory one given by (2.1)+(2.3) down to IIB. This introduces a new isometry in the action, which will have an associated abelian Killing vector l^μ pointing in the direction along which we performed the T-duality (say z). Since in the

[13] In any case, from (5.2), one can see that the 4-form does not couple to the D3 brane worldvolume theory, since the RR would be coupling would be $P[C^{(4)}] = \partial_i x^{\nu_1} \partial_j x^{\nu_2} \partial_k x^{\nu_3} \partial_l x^{\nu_4} C_{\nu_1\nu_2\nu_3\nu_4} \epsilon^{ijkl}$. Then, it is easy to see that this gives no contribution with the particular form potential in (5.2).

background we are considering we have a 4-form potential, we will keep non-vanishing couplings to $C^{(4)}$. Indeed, the would-be coupling to the 4-form is achieved by contracting $C^{(4)}$ with l^μ. We see therefore that the extra isometry is playing a key role. The final type-IIB action with coupling to the 4-form potential is given by:[14]

$$S^{BI} = -T_0 \int d\tau \ \mathrm{STr} \left\{ k^{-1} \sqrt{-P[E_{00} + E_{0i}(Q^{-1} - \delta)^i_k E^{kj} E_{j0}] det(Q^i_j)} \right\} \ , \tag{5.3}$$

where now

$$E_{\mu\nu} = g_{\mu\nu} - k^{-2} k_\mu k_\nu - l^{-2} l_\mu l_\nu - k^{-1} l^{-1} e^\phi (i_k i_l C^{(4)})_{\mu\nu} \tag{5.4}$$

$$Q^i_j = \delta^i_j + i[X^i, X^k] e^{-\phi} k l E_{kj} \tag{5.5}$$

Here k^μ is the Killing vector pointing along the direction of propagation of the gravitons, $k^2 = g_{\mu\nu} k^\mu k^\nu$, $k_\mu = g_{\mu\nu} k^\mu$, and the same notation applies to l^μ. Here we have also taken $g_{\mu\nu} k^\mu l^\nu = 0$, a condition that is satisfied for the background that we consider in this paper.

The Chern-Simons part of the action contains the term [13]

$$S_{CS} = T_0 \int P \left[(i_X i_X) i_l C^{(4)} \right] \tag{5.6}$$

in addition to the usual monopole coupling to a momentum operator, exactly as the first term in (2.3).

The fact that the direction of propagation appears as an isometric direction is common to all gravitons in type-II and M theories (see [8, 13]). The second isometric direction, z, is however special to the type-IIB case (see [14] for further details about the action). It is remarkable that only due to the presence of this isometric direction we can obtain dielectric couplings to higher order type-IIB RR potentials.

5.2 Dielectric gravitons wrapping the horizon in $AdS_3 \times S^3 \times T^4$

Using the action (5.3)+(5.6), we will consider dielectric gravitons wrapping the horizon in the $AdS_3 \times S^3 \times T^4$ geometry. As before, we will take the ansatz that we are at the origin of the torus. We will also assume that our configurations are moving in the φ direction of the background (5.1). Therefore we will take in the non-abelian action for gravitons $k^\mu = \delta^\mu_\varphi$.

In the action (5.3)+(5.6) there is an extra isometry with respect to the 11d action (2.1)+(2.3), which is due to the neccesary T-duality direction to arrive to a IIB construction. At first sight, once we assume the configuration to be in the origin of the T^4, the background (5.1) does not have any other U(1) isometry but φ. Nevertheless, the S^3 can be regarded as the Hopf fibering $S^1 \to S^2$. Then we see that there is an internal isometry hidden in a non-trivial way inside the 3-sphere. Going to adapted coordinates to see this fibering structure we have that the background reads:[15]

$$ds^2 = L^2 \left(-\cosh^2\chi d\tau^2 + d\chi^2 + \sinh^2\chi d\varphi^2 \right) + \left(\frac{L}{2}\right)^2 \left(dx^2 + dy^2 + dz^2 \right) + \left(\frac{L}{2}\right)^2 (d\chi_3 + A)^2 \,, \tag{5.7}$$

[14] For further details about the construction of this action, see [13].

[15] Again, we will omit the pieces living in the T^4.

where we have written the S^2 base in cartesian coordinates for latter convenience.

The A connection in (5.7) gives the necesary twist to the U(1) fiber parametrized with χ_3 to have an S^3. It can be explicitly seen in [13].

It is now natural to identify $l^\mu = \delta^\mu_{\chi_3}$. Therefore the non-commutative manifold will be not the full S^3 but the S^2 base. This construction is very similar to that in [13]. While in that reference it was used to describe microscopically the S^3 giant graviton which lives in $AdS_5 \times S^5$, here we use it to describe microscopically a configuration wrapping the S^3 horizon in $AdS_3 \times S^3 \times T^4$. In this case, although we are working with the same metric background as in [14], we need to construct a fuzzy S^3, which is very much the same situation as in [13].

Taking as non-commutative anstaz the same as (3.11)

$$X^i = \frac{1}{\sqrt{C_2}} J^i \tag{5.8}$$

it is straightforward to see that the hamiltonian for this configuration (again computed as minus the lagrangian) reads:

$$H = NT_0 \frac{\cosh\chi}{\sinh\chi} \sqrt{1 + \frac{(Q_1 Q_2)^2}{16C_2} \sinh^2\chi} \ . \tag{5.9}$$

Here we have made use of the symmetrized trace and taken the suitable large-N limit as in the previous cases.

Minimizing with respect to χ we see a minimum at

$$\sinh^2\chi = \frac{4\sqrt{C_2}}{Q_1 Q_2} \ , \tag{5.10}$$

whose energy is

$$E = NT_0 + \frac{N}{4\sqrt{C_2}} T_0 Q_1 Q_2. \tag{5.11}$$

5.3 D3 brane wrapping the horizon in $AdS_3 \times S^3 \times T^4$

Exactly as we did before, we can have an effective description of this configuration in terms of a D3 brane wrapping the horizon if we take sufficiently large-N in the microscopical configuration.

Assuming static gauge for a D3 brane sitting at the origin of the torus and wrapping the S^3 with non-zero velocity in the φ direction of (5.1) it is easy to see that the action is:

$$S = -T_3 L^4 \Omega_3 \int d\tau \ \sqrt{\cosh^2\chi - \sinh^2\chi\dot{\varphi}^2} \ . \tag{5.12}$$

Here Ω_3 stands for the volume of a unit 3-sphere, while T_3 is the tension of the 3-brane, whose relation with T_0 can be seen in [13].

Since φ is a cyclic coordinate, we can switch to hamiltonian formalism in terms of its canonically conjugated and conserved momentum P. The energy reads:

$$E = P \frac{\cosh\chi}{\sinh\chi} \sqrt{1 + \frac{T_0^2 (Q_1 Q_2)^2}{16P^2} \sinh^2\chi} \ . \tag{5.13}$$

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Comparing (5.13) with (5.9) we see that there is a perfect agreement once we take into account that $NT_0 = P$ by construction and that in the large-N limit when both descriptions are expected to coincide $N \sim \sqrt{C_2}$.

Minimizing (5.13), we get a minimum located at

$$\sinh^2 \chi = \frac{4P}{T_0 Q_1 Q_2} \ , \tag{5.14}$$

for which the corresponding energy is

$$E = P + \frac{T_0}{4} Q_1 Q_2 \ . \tag{5.15}$$

As we see, in the large-N limit, the macroscopical expressions coincide with their microscopical counterparts.

Again, since our configurations wrap non-contractible cycles, they are stable against collapse to a point. Since one can show that the energy has only one extremum (which is an absolute minimum) where our configurations sit, we conclude that they must be stable.

6. On the ansatz of zero momentum in the internal manifold

So far we have been considering the ansatz that our configurations are static in the internal manifold (i.e. the torus). We will return now to this point to see explicitly how this is a consistent ansatz. For this purpose we note that the general form of the action for both the M-theory and IIA macroscopic[16] configurations if we allow the brane to move in the torus (whose coordinates are generically denoted with y) is

$$S = \int d\tau \ L = \int d\tau \left\{ -F\sqrt{G - \dot{y}^2} + C(y)\dot{y} \right\}, \tag{6.1}$$

where F and G are the suitable functions in each particular case of the charges and (if in type-IIB or M-theory) the velocity in the cycle contained in the AdS; and $C(y)$ stands for the appropriate function coming from the coupling to the form potential, which in general could depend on the torus coordinates. Notice that we have integrated the dependence in the volume of the brane, and thus the action is analogous to the action for a charged particle in a magnetic field.

If we vary this action with respect to y we have that the equations of motion are of the form

$$\frac{\partial C(y)}{\partial y}\dot{y} + \frac{\partial}{\partial \tau}\left(\frac{F\dot{y}}{\sqrt{G - \dot{y}^2}} - C(y) \right) = 0 \ . \tag{6.2}$$

From (6.2) we see that once we assume $y = constant$ the equations of motion are trivially satisfied (recall that $C(y = const) = constant$), showing that our ansatz is consistent. Furthermore, we can compute the conjugated momentum to $\dot{y}$, which reads

[16] For the sake of simplicity, we will think in terms of the macroscopic description, although we could equally consider it from the microscopic point of view.

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$$\Pi = \frac{F\dot{y}}{\sqrt{G-\dot{y}^2}} - C(y) \ . \tag{6.3}$$

If we replace the ansatz of constant y we have that

$$\Pi = -C \tag{6.4}$$

But as can be seen in [6], this identification is like restricting ourselves to the lowest energy level, which is indeed in a Lowest Landau Level, as was pointed out in ([6]).

7. Conclusions

We have provided a microscopical description for some of the configurations of branes wrapping black hole horizons of [6] in terms of gravitational waves (gravitons) expanding to macroscopical configurations due to dielectric effect. Interestingly, this dielectric effect is caused not by form flux but only by gravitational effects.

The gravitational dielectric effect was studied in [10]. There it was argued that a gravitational dielectric effect for D-branes should exist in non-trivial backgrounds for time-dependent configurations. In our case we are considering gravitational waves, which carry by construction momentum. This means that our configurations are in a sense time dependent (although at the level of the effective action for the gravitational waves we do not see it, since the momentum is added as NT_0 by construction). We believe that the configurations presented in this paper are nice examples of the ones proposed in [10]. In any case, we should point that, contrary to the configurations analyzed in [10], ours are wrapping non-contractible cycles. This means that the pointlike solution is absent in this case. In any case, the construction presented here suggest that the philosophy in [10] that dielectric gravitational effect is still intact, in the sense that due to a non-trivial background, objects with non-zero velocity in a non-trivial background can polarize into a higher dimensional object, which is, for a large number of gravitons, effectively described as a macroscopic brane.

As mentioned in some points, the fact that our configurations wrap non-contractible cycles together with the fact that they sit in the lowest energy level in the internal torus indicates that the only possible instability is in the position in AdS. In general, simply by plotting the energies, it is easy to see that the extrema where our configurations sit are the absolute minima of the corresponding energy, and that any fluctuation would have more energy. In the type-IIA case we can even explicitly see that the position of the minimum is related to the one half BPS condition, so at the end of the day it is SUSY what ensures the stability.

The main interest of these configurations is that they could be of some help when understanding the black hole entropy, in very much of the same spirit of what was proposed in [1]. Since in the most general case the branes carry momentum in the internal manifold, there is a coupling to the magnetic potential, which becomes effectively a 2-form field strength on the spherical part of the geometry. Then, the lowest energy branes wrapping the horizon (which only contribute to net D0 or momentum charge) are in the Lowest

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Landau Level of the construction. As shown in [1], in the $AdS_2 \times S^2$ black hole the degeneration of the Lowest Landau Levels matches with the first order entropy formula for the black hole.

In this paper, we have concentrated on a limited class of such branes wrapping the horizon, namely the ones with zero momentum in the internal manifold (which are, as we have so far seen, precisely the lowest energy ones), giving a successful microscopical description of them. Although we have not presented it here, the extension to the most general case with momentum in the internal manifold is straightforward, but hides in a sense the purely gravitational dielectric effect responsible of these configurations. In any case, as we have shown, the ansatz of static branes in the torus is fully consistent and implies that our brane is indeed in the Lowest Landau Level.

Our main aim was to give a microscopical description of the branes in [6] in terms of gravitational waves. Therefore, in order to have agreement with the macroscopical description and in very much of the same spirit in [5], we considered a large-N limit. In any case, nothing forbids us to consider the finite N version, where our description is fully non-commutative.[17] Since this brane is wrapping the horizon, it seems that it becomes in some sense non-commutative. Although we cannot claim that our branes capture all the degrees of freedom of the horizon, it seems that since they wrap it and at finite N they are non-commutative, the horizon must be effectively non-commutative. Focusing in the IIA case, where there is a better understanding, the conjecture in [2] suggests that the black hole should be understood in terms of fixed magnetic charges and a weighted ensemble of electric charges, which led to the AdS_2/CFT_1 proposal in [1]. In that reference, using this conjecture, they computed the leading order terms in the black hole entropy, finding an agreement with the macroscopic area law. It is argued that the main contribution to the entropy comes from the degeneracy of D0 branes expanding to the S^2. As we have so far seen, this branes indeed come with a big degeneracy since they are in Lowest Landau Levels (which are discrete). This is computed assuming a large number of electric charges, i.e. zero branes. In such a situation the expanded D0 configuration becomes effectively (exactly as we have seen) a smooth D2 brane for which counting the degeneracy is easier. But indeed, as pointed in [1], there should be $\frac{1}{N}$ corrections to the entropy. In such a finite-N situation, the expanded D0 system would be no-more an abelian D2, and effectively the horizon should become non-commutative.

We have considered the black string in M-theory, whose near horizon geometry admits a stable brane wrapping the horizon, as pointed in [6]. By using the theory for coincident gravitons in [8], we were able to give a microscopical description which, as expected, overlaps with the macroscopical one when the number of gravitons is large. The black string in 11 dimensions is directly related to the type-IIA AdS_2. Upon reduction of the 11d background we arrive to a geometry whose near horizon limit is $AdS_2 \times S^2 \times T^6$. Since the direction in which we are reducing is closely related to the one in which the 11 dimensional gravitons which blow up to the S^2 wrapping the horizon in $AdS_3 \times S^2 \times T^6$

[17] Indeed since very recently it is technically possible to compute the symmetrized trace at finite N. See [21].

are moving, we expect to have a similar phenomenon in IIA, but this time in terms of dielectric D0 branes. Indeed this was analyzed in a series of papers ([2, 4, 3]) which led to a proposal for the AdS_2/CFT_1 duality in [1]. We have analyzed a limited class for such branes, which are the ones which sit at the origin of the T^6 (precisely due to this we could change the T^6 for a generic CY). We have seen that the stability of the D0 brane system is not due to a standard Myers effect, since there is no coupling to the 3-form. In turn, there is a contribution from the CS action due to a monopole coupling of each D0 to the 1-form. As we have argumented, when regarded from the 11-dimensional point of view, this is inherited from the fact that in 11 dimensions the configuration is stable only due to gravitational dielectric effect.

We have also considered the IIB $AdS_3 \times S^3 \times T^4$ background. Type-IIB backgrounds have been considered in the giant graviton literature in [13] and [14]. In those references the microscopical theory for IIB gravitons was constructed. In this case an extra isometry is needed in the non-abelian version due to the necesary T-duality in order to go from IIA to IIB. This isometry played an important role, since due to its presence we were able to couple in the $AdS_5 \times S^5$ case the 4-form potential to the 1-dimensional worldvolume of the gravitational waves ([13]). In [14], the case of the giant graviton in $AdS_3 \times S^3 \times T^4$ was analyzed. In that background there was a 2-form RR potential, and therefore, in order to couple it, the presence of an extra T-duality scalar was needed (see [14]). This T-duality extra scalar played a key role in this giant graviton, since it represented the length of a non-commutative cylinder whose sections are the physical giant gravitons (see [14] for details). In the case at hand now we have a different situation, since there is no 2-form but 4-form with entries in the torus. Therefore, once we take the ansatz that the configuration is static in the torus, the RR potential plays no role. In any case, since we are in type-IIB, we have the extra isometry l in the action. It is then natural to regard $S^3 \sim S^1 \to S^2$, and identify the S^1 fiber with l. Therefore, although in $AdS_3 \times S^3 \times T^4$, this construction is more similar to that of the giant in $AdS_5 \times S^5$ of [13]. In any case, we see that the extra isometry required by T-duality is playing a key role, since it naturally allows us to split the S^1 fiber from the S^2, which is the only pure non-abelian part of the S^3.

It is instructive to analyze a configuration in which we had the brane wrapping the horizon with no velocity in a cycle contained in the AdS. Then, the brane would tend to collapse to the origin of the AdS, since the AdS throat would pull it there. The presence of momentum in the configuration compensates this collapse and renders the brane stable far from the origin of the AdS. From this point of view, in the type-IIA case, the coupling to $C^{(1)}$ responsible from the stability of the brane can be seen as the analogue to the "centrifugal force" on each gravitational wave in the 11d configuration. Therefore, in IIA, the configuration is stable not due to a dielectric coupling, but due to a monopolar coupling which represents in a sense the force on each D0 brane.

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